RRB-JE 2024

Railway Recruitment Board

Junior Engineer Examination

Electronics Engineering

Communication Engineering, Data Communication and Network

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY PUBLICATIONS

Communication Engineering, Data Communication and Network

Contents

Chapter 1

Random Variable & Noise

1.1 Theory of Random Variable and Noise

- A waveform which can be expressed, at least in principle, as an explicit function of time is called a deterministic signal. Such signals imply complete certainty about their values at any instant of time. The signals encountered in communication systems, on the other hand, in many cases are unpredictable. In fact, information is related to uncertainty.
- The unwanted noise signals which perturb information signal transmitted over the channel are also unpredictable. Both the unpredictable signals and noise waveforms are examples of random process. Although random processes are not predictable, they are completely unpredictable either.
- It is generally possible to predict the future performance of a random process with a certain probability of being correct. In this chapter we shall discuss some essential ideas of probability theory.
- The discussion will be confined to those aspects of random processes which find application in communication system analysis.

1.2 Random variable (x)

- A random variable is a numerically valued function defined over a sample space.

OR

- The mapping function that assigns a number to each outcomes is called a random variable (X).

OR

- The random variable is used to signify a rule by which a real number is assigned to each possible outcomes of an experiment.

Discrete Random Variable (DRV)

- Here 'X' takes only a countable number of distinct values.
- Example:
  Rolling of dice = {1, 2, 3, 4, 5, 6}
  tossing of coins = {H, T} $\Rightarrow$ {1, 0}

Contineous Random Variable (CRV)

- Here 'X' can take any value and output are within the finite period.
- **Example:** Noise voltage generated in electronic amplifier.

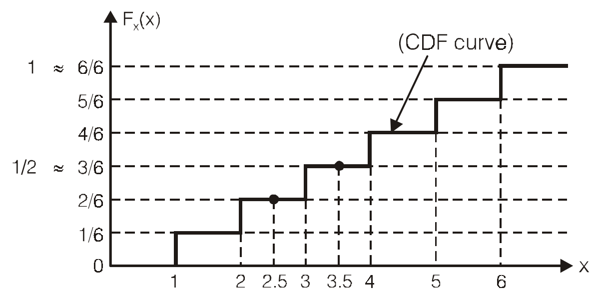
1.3 Cumulative Distribution Function (CDF)

- **'CDF'** is the probability of all possible values in the total available range.
- CDF = $F_x(x) = P(X \le x)$; $-\infty \le x \le \infty$ where, x is called Dummy variables.

Example:

Rolling of a dice

$\Rightarrow$ $S = \{1, 2, 3, 4, 5, 6\}$



$\because$ $F_x(x) = P(X \le x)$

For, $P(X \le 2.5) = \dfrac{2}{6}$ as $x = 2.5$

$P(X \le 0.9) = 0$ as $x = 0.9$

$P(X \le 6) = 1$ as $x = 6$

Properties of CDF

- $0 \le F_x(x) \le 1$
- $F_x(-\infty)$ i.e. $P(X \le -\infty)$ = no possible events $\Rightarrow 0$
- $F_x(\infty)$ i.e. $P(X \le \infty)$ = all possible events $\Rightarrow 1$
- $F_x(x_1) \le F_x(x_2)$ if $x_2 \ge x_1$, this property indicates that CDF is a monotonic non-decreasing function of x.

$\Rightarrow$ $P(X \le x_1) \le P(X \le x_2)$ if $x_2 \ge x_1$

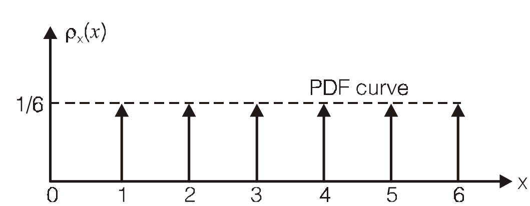
1.4 Probability Density Function (PDF)

- **PDF** is generally denoted by $f_x(x)$, $\rho_x(x)$ or $P_x(x)$ and it is defined as,

$$\boxed{PDF = f_x(x) = \frac{d}{dx}[F_x(x)]}$$

$$\boxed{PDF = \frac{d}{dx}(CDF)}$$

- It is more convenient representation for continuous random variable.
- When we differentiate the CDF-curve for rolling of a dice in figure of example then we get the PDF-curve as below in below figure.



Properties of PDF

1. $f_x(x) \geq 0$, i.e. always non-negative.
   Since CDF increases monotonically so derivative of CDF is always positive.
2. Area under the PDF-curve is always unity.

   i.e. $$\int_{-\infty}^{\infty} \rho_x(x)dx = 1$$

3. $$F_x(x) = \int_{-\infty}^{x} \rho_x(x)\,dx \quad \text{i.e.} \quad CDF = \int_{-\infty}^{x} (PDF)dx$$

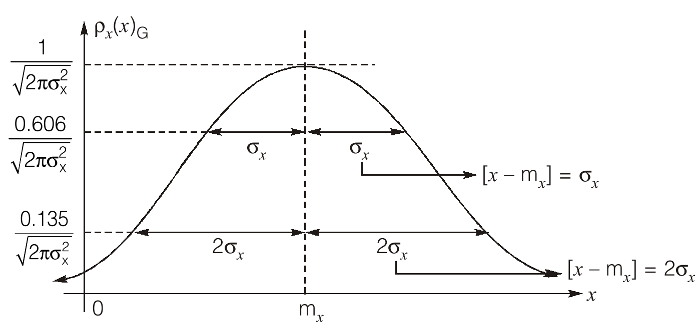
4. $P(x_1 < X < x_2) = \int_{x_1}^{x_2} \rho_x(x)dx$

1.5 Gaussian PDF

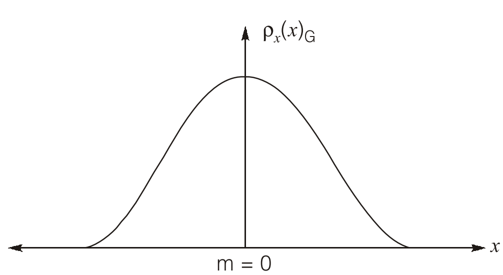
- **"Gaussian PDF"** provides a good mathematical model for various physically observed random phenomena and is given by:

$$\rho_x(x)_G = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left[\frac{-(x-m_x)^2}{2\sigma_x^2}\right] = G(mx, \sigma x^2)$$

where, m_x = mean of random variable (x)
σ_x^2 = variance of random variable (x)



- This PDF is centred about mean "m_x".



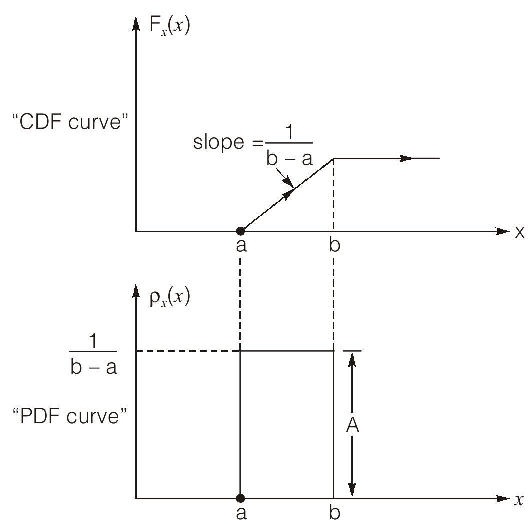
- This PDF is centred about mean $m_x = 0$

$$\therefore \quad \rho_x(x)_G = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left[\frac{-x^2}{2\sigma_x^2}\right]$$

$$= G(0, \sigma_x^2)$$

1.6 Uniform PDF

- Consider a random variable (X) whose **CDF curve** is shown below:



- For determining A,

$$\int_a^b A dx = 1$$

$$A(b-a) = 1$$

$$\boxed{A = \frac{1}{b-a}}$$

- The **"PDF-curve"** above is **PDF** of a uniformly distributed random variable (X).

1.7 Statistical Averages

Average value/Mean value/1st Moment

- Mean is a measure of where distribution is centred.
- It is denoted by $\overline{X}$ or m_x or E [X] and is given by:

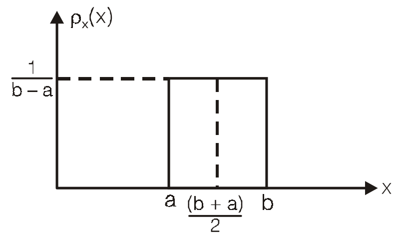
E [X] = expectation of random variable (X) = $m_x = \overline{X}$

$$\boxed{E[X] = \int_{-\infty}^{\infty} x . f_x(x) dx}$$ for CRV.

Similarly,

$$E[X] = m_x = \bar{X} = \sum_{i=1}^{N} x_i f_x(x_i)$$ for DRV.

- For an uniformly distributed random variable (x)



$\Rightarrow$
$$E[X] = \int_a^b \frac{1}{(b-a)} x\,dx = \frac{1}{2}(b+a)$$

Mean Square Value/ 2nd Moment

- It is denoted by $E[X^2]$

then,
$$E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_x(x)\,dx$$

- From the above figure we have,

$$E[X^2] = \frac{b^2 + ab + a^2}{3}$$

nth Moments

- The n^{th} moments of any random variable (X) may be defined as the mean value of X^n.
- It is denoted by $E[X^n]$ and is given by,

$$E[X^n] = \int_{-\infty}^{\infty} x^n f_x(x)\,dx$$

NOTE:

If $n = 1 \Rightarrow$ Mean value
$n = 2 \Rightarrow$ Mean square value

2nd Central moment/Variance (σ_x^2)

- The n^{th} central moment may be defined as:

$$E[(X - m_x)^n] = \int_{-\infty}^{\infty} (x - m_x)^n f_x(x)\,dx$$

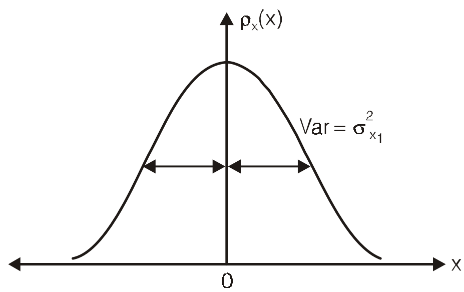
- The 2^{nd} central moment for n = 2 is called the "Variance" of random variable (X).

i.e.,
$$E[(X - m_x)^2] = \int_{-\infty}^{\infty} (x - m_x)^2 f_x(x)\,dx$$

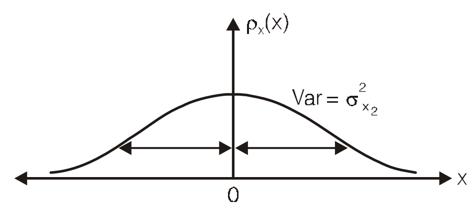
also written as

$$\text{Variance }[X] = \sigma_x^{\,2} = \text{Var}(X) = \int_{-\infty}^{\infty} (x - m_x)^2 \, f_x(x)\,dx$$

- "Variance" is referred to as the measure of the spread of distribution about its mean. The lesser the spread, the smaller is the "Variance".



(Less spread)



(More spread)

∴ $$\sigma_{x_2}^2 > \sigma_{x_1}^2$$

- From the above figure we have,

$$\text{Variance }(X) = \sigma_x^{\,2} = \frac{(b-a)^2}{12}$$

Relation between σx^2, mx and m_x^2 :

$$\sigma_x^2 = E[X^2] + m_x^2 - 2\, m_x . m_x$$

⇒ $$\sigma_x^2 = E[X^2] - m_x^2$$

$$\text{Variance } (\sigma_x^2) = \text{mean square value} - \text{square of mean value}$$

also, $$E[X^2] = \sigma_x^2 + m_x^2$$

$$m_x = \text{D.C. component}$$

$$\text{Total power} = \text{A.C. power} + \text{D.C. power}$$

$$\text{SD} = \sqrt{\sigma_x^2} = \sigma_x = \sqrt{E[X^2] - m_x^2}$$

$$\text{True r.m.s. value} = \sqrt{\text{Power}_{\text{total}}}$$

NOTE:

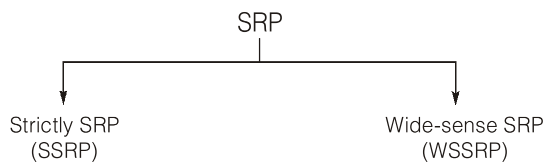
Standard deviation = S.D. = $\sqrt{\text{Variance}}$.

1.8 Random Process $X(t)$

- Time-domain representation of random variable is called **"random process** (R.P.)" $X(t)$.

Stationary Random Process (SRP)

- Its statistical properties i.e. m_x, σ^2_x, $E[X^2]$ etc are not the function of time 't'.



Strictly SRP (SSRP)

⇒ It $X(t)$ and $X(t-a)$ have properties i.e. (m_x, PDFs, $R_x(t)$, σ^2_x etc.) for all α, then $X(t)$ is called "SSRP".

Wide-senes SRP (WSSRP)

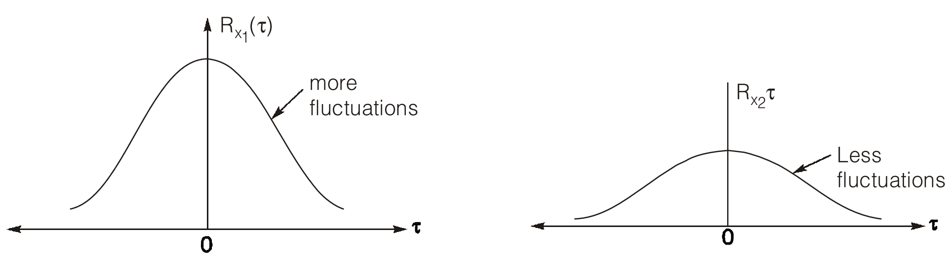
⇒ It only **"mean"** (m_x) and "Auto statistical correlation function" $\mathbf{R_x(t)}$ are stationary i.e. not a function of time, then $X(t)$ is called **"WSSRP"**.

1.9 Auto-Correlation Function (ACF)

- It is defined as,

$$\boxed{R_X(\tau) = E[X(t)\cdot X(t+\tau)]}\ ; \quad -\infty \le t \le \infty$$

- The physical significance of "ACF" is that it provides the means of describing the interdependence of random process, $X(t)$ at time **"τ second"** apart.



(Rapidly varying R.P.) (Slowly varying R.P.)

- At $\tau = 0$ the ACF we get,

$$\boxed{R_X(0) = E[X(t)\cdot X(t)] = E[X^2(t)]}$$

⇓

mean square value

- For a R.P $X(t) = A\cos(2\pi f_c \tau + \theta)$, where θ is a uniform random variable between 0 and 2π, the

$$\boxed{ACF = R_X(\tau) = \frac{A^2}{2}\cos 2\pi f_c \tau}$$

Properties of ACF

- It is always an even function of 'τ'.

  i.e. $$\boxed{R_X(\tau) = R_X(-\tau)}$$

- $R_X(\tau)$ is having maximum value at $\tau = 0$

  i.e. $$\boxed{R_X(0) \ge |R_X(\tau)|}$$

- The mean square value of the process may be obtained from $R_X(\tau)$ by simply putting $\tau = 0$.

- For a mean $m_x = 0$, the variance σ_x^2 is equal to $R_X(0)$.

  i.e. $$\boxed{\sigma_x^2 = R_X(0)}$$

  $\Rightarrow$ $$\boxed{\text{Variance} = \text{mean square value}}$$

1.10 Ergodic Random Process

- A random process is said to be Ergodic if time-average of a random process becomes equal to the mean of a random process i.e.

$$\boxed{\frac{1}{2T}\int_{-T}^{T} x(t)\,dt = m_x}$$

time-average 'ACF' becomes equal to ensemble or "$R_X(\tau)$" i.e.,

$$\boxed{\frac{1}{2T}\int_{-T}^{T} x(t)\cdot x(t+\tau)\,dt = R_X(\tau)}$$

- An Ergodic process is stationary, but a stationary process is not necessarily ergodic.

Properties of Ergodic Random Process

- The ensemble mean **"m_x"** gives the **D.C. component** of a random process $X(t)$.

  i.e. $$\boxed{m_x = \frac{1}{2T}\int_{-T}^{T} x(t)\,dt = \text{D.C. value}}$$

- The square of the mean value i.e. m_x^2 is the D.C. power of $x(t)$.
- The 2nd moment of mean square value or $E[X^2(t)]$ of random process $X(t)$ is the total average power of the random process $X(t)$.

  i.e. $$\boxed{\begin{array}{l} R_X(\tau) = E[X^2(t)] = \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt \\ \text{at } \tau = 0 \end{array}}$$

  $\Downarrow$

  average power

$$\boxed{\text{Total average power} = \text{AC power} + \text{DC power}}$$

$$\boxed{E[X^2] = \sigma_x^2 + m_x^2}$$

1.11 Wiener-Khinchine Theorem

- This theorem states that:
  "ACF and PSD (Power Spectral Density) forms the Fourier transform pair".

- "PSD" = $\frac{\text{Power}}{\text{B.W.}}$; unit → $\frac{\text{Watt}}{\text{Hz}}$
- According to theorem,

$$\boxed{\text{PSD} = \text{F.T. of ACF}}$$

i.e.

$$\boxed{S_X(\omega) = \int_{-\infty}^{\infty} R_X(\tau)\, e^{-j\omega\tau} d\tau}$$

∴

$$\boxed{R_X(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)\, e^{j\omega\tau} d\omega}$$

⇒

$$\boxed{\text{ACF} = \text{Inverse F.T. of PSD}}$$

- Also, it can be written as,

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f\tau} d\tau$$

∴

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) e^{j2\pi f\tau} df$$

Properties of PSD

- Zero frequency of PSD = total area under the ACF curve

i.e.

$$\boxed{S_X(0) = \int_{-\infty}^{\infty} R_X(\tau) d\tau}$$

- The "mean square value" of WSSRP, $X(t)$ equals to the total area under the graph of PSD. (For this we put $\tau = 0$)

i.e.

$$\boxed{R_X(0) = E[X^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega) d\omega}$$

also,

$$\boxed{R_X(0) = E[X^2(t)] = \int_{-\infty}^{\infty} S_X(f) df}$$

- PSD is always non-negative i.e.,

$$\boxed{S_X(\omega) \geq 0}$$

- PSD is an even function of frequency.

i.e.,

$$\boxed{S_X(\omega) = S_X(-\omega)}$$

OR

$$\boxed{S_X(f) = S_X(-f)}$$

So, we can also write PSD of Y(t) as

$$S_y(\omega) = \frac{1}{4}\left[S_x(\omega - \omega_c) + S_x(\omega + \omega_c)\right]$$

or

$$S_y(f) = \frac{1}{4}\left[S_x(f - f_c) + S_x(f + f_c)\right]$$

- Given the power spectral density of noise waveform n(t), the power spectral spectral density of n(t) $\cos 2\pi f_c t$ is arrived at as follows:

  $\Rightarrow$ Divide $S_n(f)$ by 4, shift the divided plot to the left by f_c, to the right by f_c and add the two shifted plots.

Student's Assignments

Q.1 The RMS thermal noise voltages of three resistors individually are E_1, E_2 and E_3. If these resistors are connected in series, the total noise voltage is given by

(a) $E_1 + E_2 + E3$

(b) $\left(\sqrt{E_1} + \sqrt{E_2} + \sqrt{E_3}\right)^2$

(c) $\sqrt{(E_1^2 + E_2^2 + E_3^2)}$

(d) $(E_1 E_2 E_3)^{1/3}$

Q.2 The mean square value of the shot noise current

(a) varies inversely as average current

(b) is independent of average current

(c) varies as $\sqrt{\text{average current}}$

(d) varies directly as average current

Q.3 Gaussian probability density function is given by $\frac{k}{\alpha} e^{\frac{-(x-m)^2}{2\sigma^2}}$. The value of k is

(a) $\frac{\alpha}{\sigma\sqrt{2\pi}}$ (b) $\frac{1}{\sqrt{2\pi}}$

(c) $\frac{\sigma\alpha}{\sqrt{2\pi}}$ (d) $2\sqrt{\pi\alpha}$

Q.4 Band-limited White Gaussian Noise means that the spectral density of the noise

(a) is constant over a given band and has any value elsewhere

(b) is constant over a given band and has zero value elsewhere

(c) is zero over the given band and constant elsewhere

(d) varies over a given band and has zero value elsewhere

Q.5 If E denotes expectation, the variance of a random variable X is given by

(a) $E[X^2] - E^2[X]$ (b) $E[X^2] + E^2[X]$

(c) $E[X^2]$ (d) $E^2[X]$

Q.6 The PDF of a Gaussian random variable X is given by

$$\rho X(X) = \frac{1}{3\sqrt{2\pi}} e^{\frac{-(x-4)^2}{18}}$$

The probability of the event $\{X = 4\}$ is

(a) 1/2 (b) $\frac{1}{3\sqrt{2\pi}}$

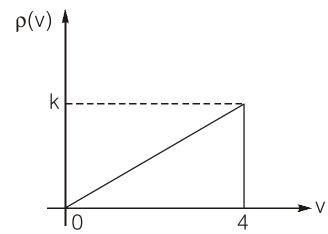
(c) 0 (d) 1/4

Q.7 The rms noise voltage of an amplifier due to an input resistor is 20 microvolts in a bandwidth of 2 MHz. If the operating bandwidth is doubled, the noise voltage will become (rounded to nearest whole number)

(a) 40 microvolts (b) 35 microvolts

(c) 30 microvolts (d) 20 microvolts

Q.8 An output of a communication channel is a random variable v with the probability density function as shown in the figure. The mean square value of v is



(a) 4 (b) 6
(c) 8 (d) 9

Q.9 The noise temperature is about 4306 K. The noise figure will be
(a) 3 dB (b) 6 dB
(c) 12 dB (d) 24 dB

STUDENT'S ASSIGNMENTS ANSWER KEY

1. (b)	2. (d)	3. (a)	4. (b)	5. (a)
6. (c)	7. (c)	8. (c)	9. (c)	

STUDENT'S ASSIGNMENTS EXPLANATIONS

6. (c)

Probability of a Gaussian R.V 'X' is not defined at the single point.

So here, $\boxed{\rho_x(x) = 0}$

9. (c)

$$\text{Noise figure} = F = 1 + \frac{T_{eq}}{T_0} = 1 + \left(\frac{4306}{290}\right)$$

(F) in dB = 12 dB

■■■■